PMM U.S.S.R.,Vol.45,pp.291-292
0021-8928/82/2 0291 $7.50/0

UDC 536.12

ON DANILOVSKAIA PROBLEM FOR THE CASE OF A BOUNDARY OF AN ELASTIC HALF-SPACE MOVING WITH CONSTANT VELOCITY*

B.V. SEMKIN and A.V. IUSHIN

In the course of using electric discharges and electrically exploded conductors and other processes characterized by high temperatures, one encounters the dynamic problems of thermal stresses with moving boundaries. The present paper offers a solution of a one-dimensional problem of thermal stresses appearing in an elastic half-space as a result of sudden heating of its boundary, for the case when the boundary moves with constant velocity v.

For a homogeneous, isotropic half-space the system of equations of thermal stresses has the form /1/

$$c^2 \frac{\partial^2 \sigma}{\partial x^2} - \frac{\partial^2 \sigma}{\partial t^2} = S \frac{\partial^2 T}{\partial t^2}, \quad S = \alpha_t (3\lambda + 2\mu) \tag{1}$$

$$a^2 \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}, \quad c^2 = \frac{2\mu + \lambda}{\rho} \tag{2}$$

Here σ is the stress, α_t is the thermal expansion coefficient, λ and μ are the isothermal Lamé constants, T is temperature, c^2 is the square of the speed of sound and a^2 is the heat conduction coefficient.

The initial and boundary conditions in the above formulation are

$$\sigma(x, 0) = 0, \quad \partial\sigma(x, 0)/\partial t = 0, \quad \sigma(vt, t) = 0 \tag{3}$$

$$T(x, 0) = 0, \quad T(vt, t) = T_0, \quad x > vt \tag{4}$$

A solution of the equation of heat conduction (2) with initial and boundary conditions (4) can be easily obtained by passing to a moving frame of reference $y = x - vt,\ t = t$, and has the form

$$T = \frac{1}{2} T_0 \left[\operatorname{erfc}\left(\frac{x}{2a\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{x}{2a\sqrt{t}} - \frac{v}{a}\sqrt{t}\right) \exp\left[-\frac{v}{a^2}(x - vt)\right]\right] \tag{5}$$

A solution of (1) with conditions (3) and (5) taken into account, can be conveniently obtained using the Laplace transformation. Assuming that σ is bounded as $x \to \infty$, we obtain the following Laplace transform for $\bar{\sigma}$:

$$\sigma = \frac{ST_0}{2}\left[\frac{\sqrt{p}}{\left(\sqrt{p} - \frac{v}{a}\right)\left(\frac{c^2}{a^2} - p\right)} + \frac{1}{\frac{c^2}{a^2} - p}\right] \exp\left(-\frac{x}{a}\sqrt{p}\right) + \exp\left(-\frac{x}{c}p\right)\bar{F}(p) \tag{6}$$

where $\bar{F}(p)$ denotes the transform of an arbitrary function. Passing in (6) to the original function and taking into account (3), we obtain /2/

$$\sigma = \frac{ST_0}{2}\left[F_1(\xi, \tau) - H\left(t - \frac{x}{c}\right) F_2(\xi, \tau)\right] \tag{7}$$

$$F_1(\xi, \tau) = \left[1 + \frac{\beta}{2(\beta - 1)}\right] e^{\tau-\xi} \operatorname{erfc}\left(\frac{\xi}{2\sqrt{\tau}} - \sqrt{\tau}\right) +$$
$$\left[1 - \frac{\beta}{2(1+\beta)}\right] e^{\tau+\xi} \operatorname{erfc}\left(\frac{\xi}{2\sqrt{\tau}} + \sqrt{\tau}\right) +$$
$$\frac{\beta^2}{\beta^2 - 1} e^{\beta(\beta\tau - \xi)} \operatorname{erfc}\left(\frac{\xi}{2\sqrt{\tau}} - \beta\sqrt{\tau}\right)$$

$$F_2(\xi, \tau) = \left[1 + \frac{\beta}{2(1 - \beta)}\right] e^{\tau-\xi} \operatorname{erfc}_+ \left[\left(1 - \frac{\beta}{2}\right)\sqrt{\frac{\tau - \xi}{1 - \beta}}\right] +$$
$$\left[1 - \frac{\beta}{2(1+\beta)}\right] e^{\frac{1+\beta}{1-\beta}(\tau-\xi)} \operatorname{erfc}\left[\left(1 + \frac{\beta}{2}\right)\sqrt{\frac{\tau - \xi}{1 - \beta}}\right] +$$

*Prikl.Matem.Mekhan.,Vol.45,pp.394-396,1981

$$\frac{\beta^2}{\beta^2-1}\,\mathrm{erfc}_+\left(\frac{\beta}{2}\sqrt{\frac{\tau-\xi}{1-\beta}}\right), \quad \mathrm{erfc}_+(x)=1+\Phi(x),$$
$$\xi=\frac{c}{a^2}x, \quad \tau=\frac{c^2}{a^2}t, \quad \beta=\frac{v}{c}$$

where $H(t)$ is the Heaviside function. When $\beta\to 0$ $(v\to 0)$, the solution (7) becomes a solution of the Danilovskaia problem /3/ (see also /1/).

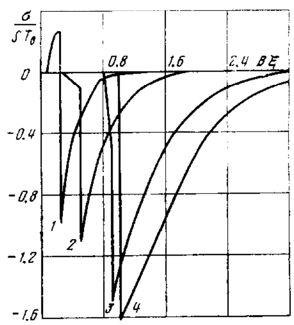

Fig.1

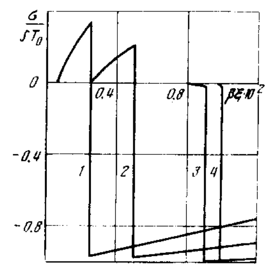

Fig.2

In the solution obtained, as well as in the Danilovskaia problem, the stresses are represented by the diffusing part of the wave $F_1(\xi,\tau)$, which appears at once in every point of the half-space, and by an elastic wave $F_2(\xi,\tau)$, which moves with velocity c. The stress undergoes a discontinuity at the elastic wave front, and the value of the jump is sT_0. It follows therefore that the motion of the boundary does not affect the magnitude of the jump.

Figure 1 uses the $\sigma/(ST_0), \beta\xi$ coordinates to depict the curves $1-4$ for $\tau=1$ obtained for β equal to 0.25, 0.5, 0.9 and 0.99. Fig.2 shows the solutions for $\tau=10^{-2}$, for the same values of β.

Positive stresses appear between the boundary of the half-space and the elastic wave front $\beta\tau<\xi<1$, and they become negative with increasing τ just as in the case of the Danilovskaia problem. The stress amplitudes however, become greater, the greater $\beta(v)$. Moreover, it is characteristic for the present problem that the stresses change their sign the earlier, the greater the velocity v of the boundary. When $\beta\to 1$, large compressive stresses appear at large τ near the thermoelastic wave front.

REFERENCES

1. NOVATSKII V., Theory of Elasticity. Moscow, MIR, 1975.

2. DITVIN V.A., and PRUDNIKOV A.P., Handbook of Operational Calculus. Moscow, Vysshaia shkola, 1965.

3. DANILOVSKAIA V.I., Thermal stresses in an elastic half-space, due to a sudden heating of its boundary. PMM Vol.14, No.3, 1950.

Translated by L.K.